Day 4 props (2nd half of Day 7, 2013)

Name: ______________________ Section: ______________________

Combining Functions

If $f(x)$ is one function, and if $g(x)$ is another function, then we can combine them in several ways.

Add functions: To compute $(f+g)(x) = f(x) + g(x)$

Pretend you are a computer!

Step 1: FIND $f(x) = a$

Step 2: FIND $g(x) = b$

Step 3: WRITE $(a)+(b)$

If ANY step fails, STOP & crash!

Therefore, $(f+g)(x)$ is defined if Both $f(x)$ and $g(x)$ are defined

If you know the domain of f and the domain of g, this lets you find the domain of $(f+g)$

Subtract functions: To compute $(f-g)(x) = f(x) - g(x)$

Step 1: FIND $f(x) = a$

Step 2: FIND $g(x) = b$

Step 3: WRITE $(a)-(b)$ & simplify

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Therefore, $(f-g)(x)$ is defined if Both $f(x)$ & $g(x)$ are defined

If you know the domain of f and the domain of g, this lets you find the domain of $(f-g)$

Multiply functions: To compute $(f \cdot g)(x) = f(x) \cdot g(x)$

Step 1: FIND $f(x) = a$

Step 2: FIND $g(x) = b$

Step 3: WRITE $(a)(b)$ & simplify

Therefore, $(f \cdot g)(x)$ is defined if Both $f(x)$ & $g(x)$ are defined

If you know the domain of f and the domain of g, this lets you find the domain of $(f \cdot g)$

Divide functions: To compute $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

WARNING: this is UNDEFINED if $g(x) = 0$

Step 1: Find $f(x) = a$

Step 2: Find $g(x) = B$

Step 3: WRITE $\left(\frac{a}{b}\right)$ & simplify

Therefore, $\left(\frac{f}{g}\right)(x)$ is defined if All 3 things are ~~[illegible]~~ true

① $f(x)$ is defined

② $g(x)$ is defined

③ $g(x) \neq 0$

If you know the domain of f and the domain of g, this lets you find the domain of $\left(\frac{f}{g}\right)$

Eg: let $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x}$

what is the domain of

$$(f+g)(x) = \frac{1}{x} + \sqrt{x} \quad ?$$

$f(x) = \frac{1}{x}$ is defined for $x \neq 0$

$g(x) = \sqrt{x}$ is defined for $x \geq 0$

$\Rightarrow$ Both are defined for $x > 0$

10 Domain of $(f+g)(x)$ is $(0, \infty)$

Eg: let $f(x) = x + 1$ and $g(x) = \sqrt{x}$

$f(x)$ is defined for all x

$g(x)$ is defined for $x \geq 0$

& $g(x) \neq 0$ for $x \neq 0$

so $\frac{f}{g}(x)$ is defined for x in $(0, \infty)$

Name: ______________________ Section: ______________________

Compose functions: To compute $(f \circ g)(x) = f\big(g(x)\big)$

only when you have an equation for f(x)

Step 1: FIND $\boxed{g(x)}$

Step 2: WRITEE $f(x)$ out, & draw a box where each x term was [redacted]

Step 3: Plug $\boxed{g(x)}$ into each x term of $f(x)$

Therefore, $(f \circ g)(x)$ is defined if BOTH $g(x)=b$ is defined AND $f(g(x))=f(b)$ is defined

If you know the domain of f and the domain of g, this lets you find the domain of $f \circ g$

E.g. Suppose $f(1) = 2$, $f(3) = 4$ and that $g(1) = 3$, and that $g(2) = 1$. Compute $(f \circ g)(1)$.

$$(f \circ g)(1) = f\big(g(1)\big)$$

Step 1

$$g(1) = 3$$

Step 3

$$= f(3)$$

$$= 4$$

$$\boxed{(f \circ g)(1) = 4}$$

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(-20)

E.g. Suppose $f(x) = x^2 + \sqrt{x} + 1$ and $g(x) = \frac{1}{x}$. Compute $(f \circ g)(x)$, and find its domain.

$$(f \circ g)(x) = f\big(g(x)\big)$$

Step 1: $g(x) = \frac{1}{x}$

$$= f\left(\boxed{\tfrac{1}{x}}\right)$$

Step 2: $f(x) = (\boxed{x})^2 + \sqrt{\boxed{x}} + 1$

Step 3

$$= \left(\boxed{\tfrac{1}{x}}\right)^2 + \sqrt{\boxed{\tfrac{1}{x}}} + 1$$

What is the domain of $(f \circ g)$?

$$= \frac{1}{x^2} + \frac{1}{\sqrt{x}} + 1$$

NOTICE:

① $g(x) = \frac{1}{x}$ is defined when $x \neq 0$

② $f(g(x)) = f\left(\frac{1}{x}\right)$ is defined when $g(x) = \frac{1}{x} \geq 0$

So when $\frac{1}{x} \geq 0$

which happens when $x \geq 0$

③ So BOTH g(x) AND f(g(x)) are defined when $x > 0$ $\Rightarrow$ Domain is $(0, \infty)$

Eg:

① Write $\sqrt{1-x^2}$ ← the inside function

as a composition of functions

② find its domain

① Notice $f(\boxed{x}) = \sqrt{\boxed{x}}$

$g(x) = 1 - x^2$

then $(f \circ g)(x) = f\big(g(x)\big) = \sqrt{g(x)} = \sqrt{1-x^2}$

(10) ② $(f \circ g)(x)$ is defined

$\Longleftrightarrow$

① $g(x)=a$ is defined

② $f(a)$ is defined

NOTICE $g(x)=a$ is always defined

and $f(a)$ is defined $\Longleftrightarrow$ $a = g(x) \geq 0$

$g(x) \geq 0$

$\Rightarrow 1 - x^2 \geq 0$

$\Rightarrow 0 \geq x^2 - 1 = (x+1)(x-1)$

$\Rightarrow$ x is in $[-1, 1]$

$x=0 \Rightarrow$ true

$x=2 \Rightarrow$ false

Step 1: = when

Step 2: undef

Step 3: fill in intervals

Inverses are functions that undo

Eg: $g(x) = x + 1$ undoes $f(x) = x - 1$

$g(x) = \sqrt{x}$ undoes $f(x) = x^2$ [redacted]
AS LONG AS $x \geq 0$

$g(x) = \frac{x}{2}$ undoes $f(x) = 2x$

Define if $g(x)$ is a function
10 that undoes $f(x)$

we call $g(x)$ the inverse of $f(x)$

and write $f^{-1}(x) = g(x)$

Eg: I If $f(x) = x - 1$

$f^{-1}(x) = x + 1$

II If $f(x) = x^2$ IS RESTRICTED TO $x \geq 0$

$f^{-1}(x) = \sqrt{x}$ [redacted]

Our Most important Inverse

<u>Define:</u> $\log_a(x)$ is the inverse of a^x

$\ln(x)$ is the inverse of e^x

<u>This means:</u>

$$\log_a(a^x) = x$$

$$a^{\log_a(x)} = x$$

<u>Eg:</u> $\log_{10}(100) = \log_{10}(10^2) = 2$

$$\log_2(8) = \log_2(2^3) = 3$$

<u>Eg:</u> $\ln\left(e^{2x+1}\right) = 2x+1$

$$e^{\ln(x^2-4)} = x^2-4$$

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to find an inverse from a graph:

Reflect across the line $y = x$

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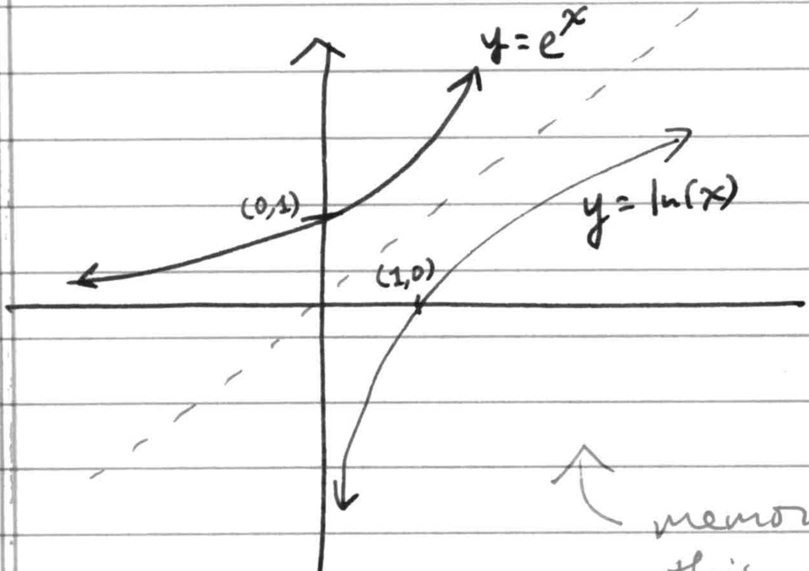

memorize this picture!